Planar Kinetics of a Rigid Body
Force and Acceleration
Outline

• Moment of Inertia
• Planar Kinetic Equations of Motion
• Equations of Motion: Translation
• Equations of Motion: Rotation about a Fixed Axis
• Equations of Motion: General Plane Motion
Moment of Inertia

• Body = Size + Shape
• Motion = Translation + Rotation
• Translation
  – F = ma
• Rotation
  – M = Iα
  – I = moment of inertia
  – I → resistance to angular acceleration
Moment of Inertia (review)

\[ I = \int m \cdot r^2 \, dm \]

\[ I = I_G + md^2 \]
\( d = \) perpendicular distance between the parallel axes

\[ I = \int_V \rho \cdot r^2 \, dV \]

\[ I = mk^2 \]
\( k = \) radius of gyration
Equation of Motions

Equation of Translational Motion

\[ \sum \vec{F} = m \vec{a}_G \]

• For motion of the body in the x-y plane

\[ \sum F_x = m(a_G)_x \]

\[ \sum F_y = m(a_G)_y \]
Equation of Rotational Motion

\[ \sum \tilde{M}_G = I_G \ddot{\alpha} \]

@ Point P

\[ \sum \tilde{M}_P = (I_G + md^2) \ddot{\alpha} \]
Equation of Motions

Rectilinear Translation

\[ \sum F_x = m(a_G)_x \]
\[ \sum F_y = m(a_G)_y \]
\[ \sum \vec{M}_G = 0 \]
Example 1

A uniform 50-kg crate rest on a horizontal surface for which the coefficient of kinetic friction is 0.2. determine the acceleration if a force of $P = 600\text{N}$ is applied to the crate.
Equation of Motions

Curvilinear Translation

\[ \Sigma F_n = m(a_n)_G = m\omega^2 r \]
\[ \Sigma F_t = m(a_t)_G = m\alpha r \]
\[ \Sigma M_G = 0 \]
The crate C has a weight of 1500 N and rests on the truck elevator for which the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration $\alpha$, starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.
Equation of Motions

Rotation about a fixed axis

\[ \sum F_n = m \left( a_n \right)_G = m \omega^2 r \]
\[ \sum F_t = m \left( a_t \right)_G = m \alpha r \]
\[ \sum M_O = \left( I_G + mr^2 \right) \alpha \]
Problem 17-53

The 80-kg disk is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.
The 20-kg roll of paper has a radius of gyration $kA = 90$ mm about an axis passing through point A. It is pin-supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of paper, determine the angular acceleration of the roll as the paper unrolls.
Problem 17-73

The disk has a mass of 20 kg and is originally spinning at the end of the strut with angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, for which the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?

$t = 3.11\text{ s} \quad F_{CB} = 193\text{ N}$
Rotation with Friction

\[
\begin{align*}
\sum F_x &= m\left(a_x\right)_G \\
\sum F_y &= m\left(a_y\right)_G \\
\sum M_P &= I_G \alpha
\end{align*}
\]

\[
\begin{align*}
P - F &= ma_G \\
N - mg &= 0 \\
Fr &= I_G \alpha
\end{align*}
\]
Friction Conditions

- Case 1: No Slipping
  \[ a_G = \alpha r \quad (F \leq \mu_s N) \]

- Case 2: Slipping
  \[ F = \mu_k N \]
Example 2

The 25kg wheel has a radius of gyration of $k_G = 0.2m$. If a 50N.m couple moment is applied to the wheel, determine the acceleration of its mass center $G$. The coefficients of static and kinetic friction between the wheel and the plane at $A$ are $\mu_s = 0.3$ and $\mu_k = 0.25$ respectively.
Problem 17-26

The 20-N bottle rests on the check-out conveyor at a grocery store. If the coefficient of static friction is 0.2, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at G.
The car accelerates uniformly from rest to 26.4 m/s in 15 seconds. If it has a weight of 19 kN and a center of gravity at G, determine the normal reaction of each wheel on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be 0.4 and 0.2.